Don't guess the spurious level

of an amplifier. The intercept-point method gives exact values with the aid of a simple nomograph.

Here is a simple way to predict accurately the spurious responses of an amplifier through bands of frequencies. The designer has to know the order and the magnitude of a response only at one frequency to be able to find all responses for any fundamental signal.

Exact knowledge of all spurious signals is essential in designs where an optimum trade-off between dynamic range and low noise is sought; for example, in receivers. This approach is increasingly preferred because of overcrowding of the frequency spectrum and the demand for versatility. When these considerations are important, the designer cannot use the conventional method—that is, shoot for lowest noise and settle for the resulting dynamic range.

The new technique makes use of the intersection of the fundamental and third-order responses on a log-log scale; therefore, it is called the intercept-point method. Besides yielding exact spurious levels, the intercept point can also be used to specify the amplifier in a more general and succinct way. For example, instead of "intermodulation products must be down 60 dB with respect to two signals at 0-dBm output," the engineer can say, "intercept point at +30 dBm."

In the past, when low noise was the primary design goal, a rule of thumb was used to predict the spurious response level, and hence the dynamic range, of amplifiers. This rule states that if two signals are present in an amplifier with an amplitude which is s dB below the 1-dB gain compression point, the third-order intermodulation products will be $2s$ dB below the signal level. However, this rule does not give accurate values for multi-stage transistor amplifiers.

Intercept point predicts 2nd and 3rd responses

A brief mathematical analysis is necessary to establish the relationship between the collector current and spurious signals. The instantaneous transistor collector current, as a function of the base current, can be expressed as:

$$i_c = i_{bo} + k_1 i_b + k_2 i_b^2 + k_3 i_b^3. \qquad (1)$$

If two signals are present at the base of the transistor, such that the base current is:

$$i_b = i_{bo} + a_1 \cos \omega_1 t + a_2 \cos \omega_2 t, \qquad (2)$$

then the power-series expansion of the collector current, with Eq. 2 substituted for the base current, will contain components that are functions of:

- The dc bias current.
- The two input signals.
- Harmonics of the two input signals.
- Intermodulation products of the two signals and/or their harmonics.

The following are the pertinent terms of this expansion:

Fundamental:

$$k_1 a_1 \cos 2\pi f_1 t$$
$$k_1 a_2 \cos 2\pi f_2 t$$

Second order:

$$k_2 a_1 a_2 \cos 2\pi (f_1 \pm f_2) t$$
$$(1/2) k_2 a_1^2 \cos 2\pi (2f_1) t$$
$$(1/2) k_2 a_2^2 \cos 2\pi (2f_2) t$$

Third order:

$$(3/4) k_3 a_1^2 a_2 \cos 2\pi (2f_1 \pm f_2)$$
$$(3/4) k_3 a_1 a_2^2 \cos 2\pi (2f_2 \pm f_1)$$

This partial list of terms reveals that:

- The fundamental responses are directly proportional to the level of the input signals.
- The second order responses are proportional to the square of the input amplitude.
- The third order responses are proportional to the cube of the input amplitude.

Note that no assumptions have been made so far. Hence, it can be concluded that a plot of each response on a log-log scale (or dB/dB scale) will be a straight line with a slope corresponding to the order of the response; i.e., the fundamental responses will have a slope of 1, the second order responses will have a slope of 2, etc. It is therefore sufficient to know the order of the response and its magnitude at one point alone in order to be able to plot the level of each response.

That point at which the fundamental response and the third order spurious responses intercept

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will be used here as the reference. It is labeled the "Intercept Point" (P_i). There are several reasons for its choice.

The first reason is, of course, that it is a point established through mathematical analysis and yields more information. Contrast it, for instance, with the 1-dB gain compression point, which reveals only the deviation from the linear at one point of a curve. The shape of this curve at any other point can only be guesswork.

Another reason for selecting the intersection of the fundamental and third-order signals is that the third-order intermodulation product is generally the one that poses the most serious problems in a case where the bandwidth of the device under consideration is less than an octave. In addition, it transpires that the second-order plot intercepts the fundamental at the same point as the third-order. Incidentally, the measurements leading to the conclusion also confirmed that the second harmonic response is 6 dB below the second-order intermodulation response, as predicted by the expanded terms—$(1/2)k_2 a_1^2 \cos 2\pi(2f_2)t$ versus $k_2 a_1 a_2 \cos 2\pi(f_1 \pm f_2)t$. The intercept point may be also used, therefore, to predict the second-order response with good accuracy.

There are some exceptions, due to the system make-up, when the second- and third-order plots do not intercept at the same point. If some technique is used that suppresses even-order spurious responses (such as a push-pull operation), the second-order intercept point may be expected to be higher than that of the third-order. This would not affect the slope, which would still be two and three, respectively.

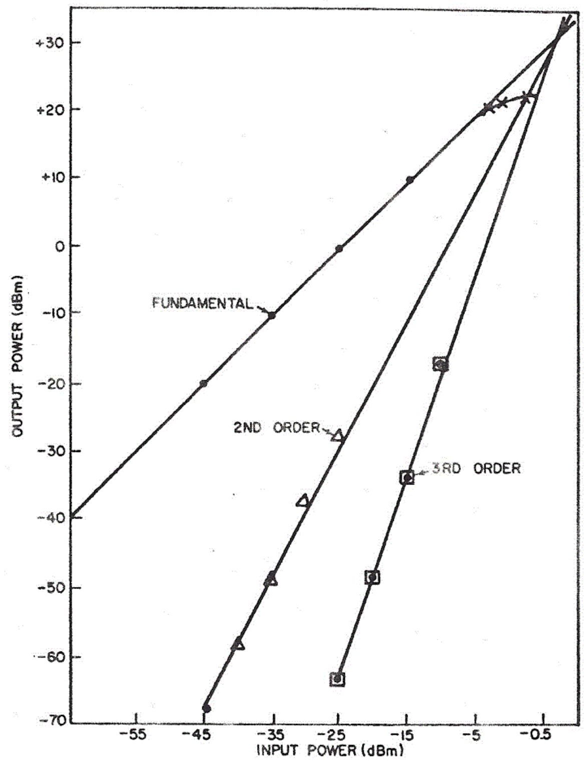
The concept of the intercept point is therefore valid for those cases, too, where the intercept for the second order does not coincide with that of the third order. However, then the intercept points for second and third order should be specified separately.

A typical set of measurements is shown in Fig. 1. The amplifier—an Avantek Model AP-20—was operated at 300 MHz. The linear portions of the responses have been extrapolated to the point where they intersect. The slope of the lines, drawn through the measured points, is in excellent agreement with the theory. The difference between predictions with the intercept-point and the 1-dB compression-point concepts is illustrated in Fig. 2, through a frequency band from 200 to 400 MHz.

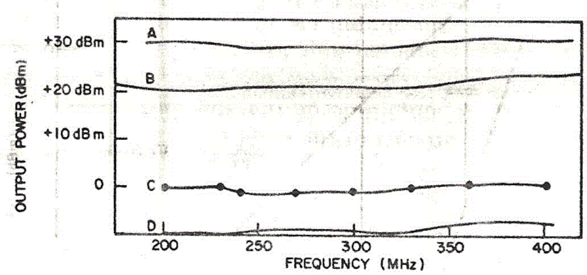
Measured values, indicated by the circled points in Fig. 2 near line C, confirm predictions based on the intercept-point method.

How to use the intercept point

With the intercept point known, it is very simple to calculate the level of spurious responses. The nomograph in Fig. 3 comes handy in the process.



1. **The plot of amplifier responses** is a set of straight lines on the log-log scale. The slope of the line depends on the order; the fundamental has a slope of 1, the second order has a slope of 2 and the third order has a slope of 3. The intersection of the fundamental and third order yields the intercept point.

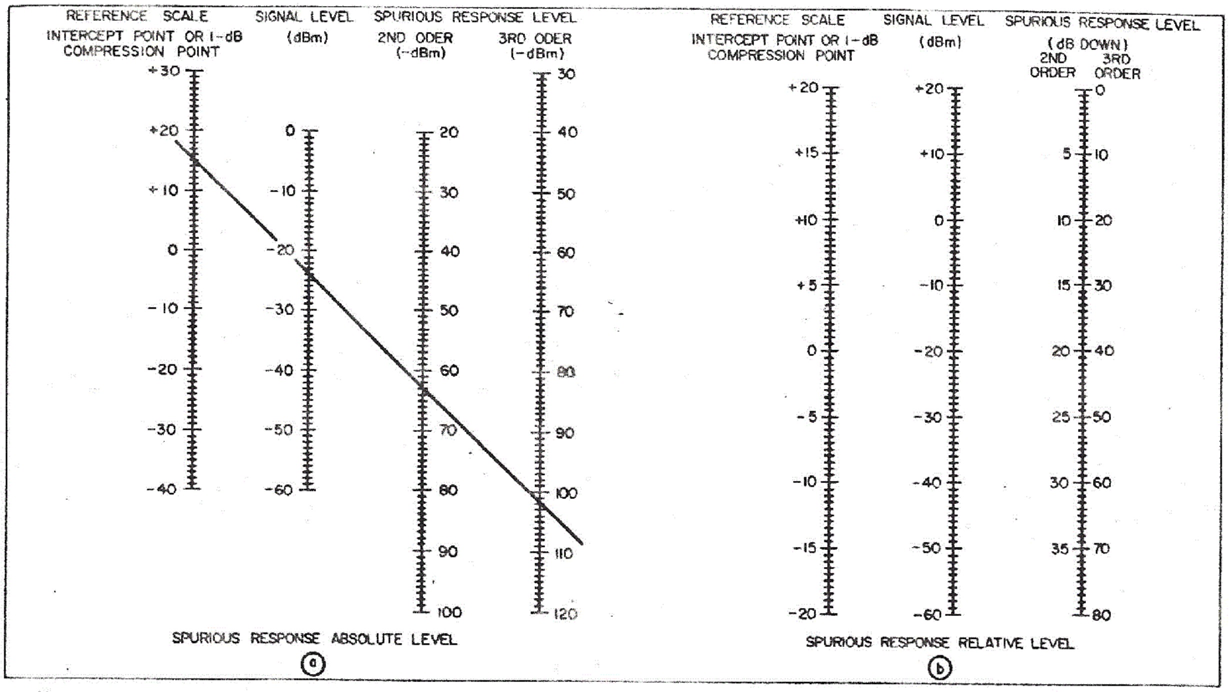


2. **A comparison of signal levels,** predicted by the intercept method (A) and with the 1-dB compression point (B) shows difference. To achieve a 3rd-order IM level 60 dB down, the signal level predicted with the intercept point (C) is about 10 dBm above that predicted with the 1-dB compression point (D). Dots on (C) are test results.

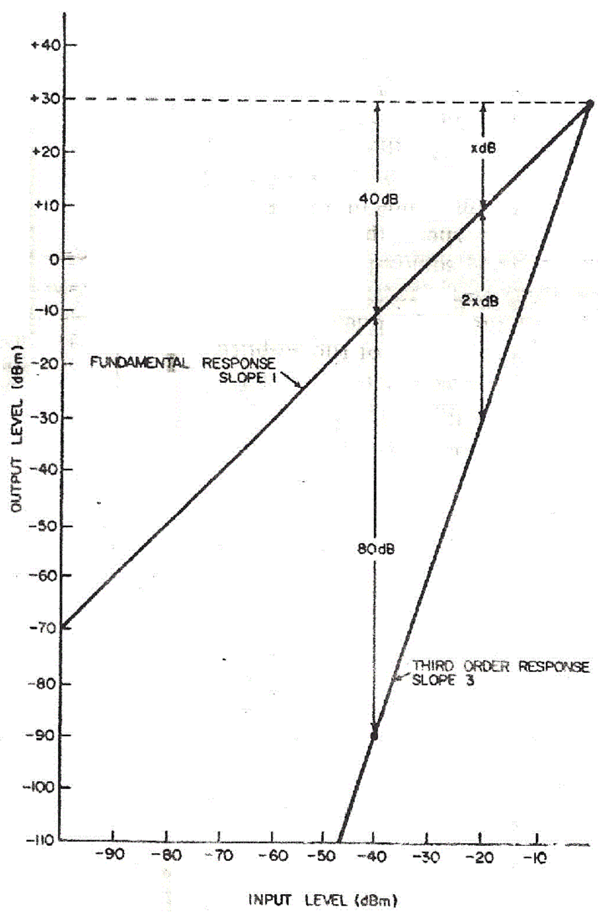
(Copies of the nomograph are available on a heavy stock paper. For your copy, circle Reader Service No. 349 on the card at the back of the magazine.)

Assume, for example, that the P_i is + 30 dBm. The output contains two equal signals of −10-dBm amplitude. What is the level of third-order intermodulation (IM) spurious responses?

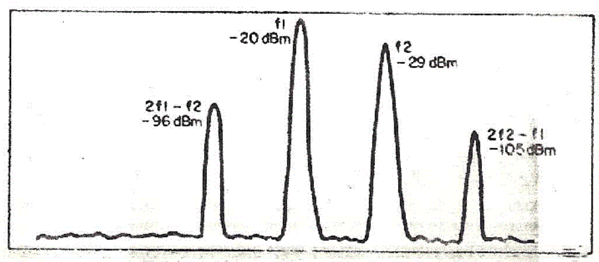
As the signal level is 40 dB below the intercept point, the third-order response, having a slope of 3, must be 80 dB below these signals, or at −90



3. Nomographs allow rapid calculations of the absolute (a) and relative (b) spurious levels of the second- and third-order intermodulation signals. The example worked out in the text is illustrated on (a).



4. The level of the third-order response can be found immediately, once the intercept point and the fundamental signal are known.



5. The third-order spurious signals, generated by two unequal inputs, will appear on a spectrum analyser as this sketch. Their amplitude can be found by establishing an equivalent amplitude for the two input signals and considering two identical signals at the new level.

dBm, as shown in Fig. 4.

Or, to tackle the same problem from another direction, assume that another set of parameters is specified: The noise figure is 3 dB and the final IF bandwidth is 5 MHz. How do you find the dynamic range of the preamplifier?

With a 3-dB noise figure and 5-MHz bandwidth, the signal-to-noise ratio becomes unity; that is, $S = N$ when:

$$-114 + 3\ \mathrm{dB(NF)} + 7\ \mathrm{dB(BW)} = -104\ \mathrm{dBm}.$$

In other words, the noise power, referred to the input of the amplifier, is −104 dBm for every 5-MHz bandwidth increment. A minimum detectable signal level may then be −101 dBm. Assuming an intercept point of +15 dBm, the total range between minimum detectable signals and the intercept point is 116 dB. Two signals, each having an amplitude of one-third the total range, will have spurious responses at the minimum detectable signal level; therefore, divide 3 into 116. Thus each of the two signals is at about 39 dB below the intercept point. From the nomogram, two signals 39 dB below the intercept point, or at −24 dBm, have third-order spurious responses at −102 dBm. The spurious-free dynamic range in this case is − 102 dBm − (− 24 dBm) − 78 dB.

The next obvious case to consider is that of two unequal signals. The two signals and their third-order IM products appear on a spectrum analyzer as shown in Fig. 5.

The two signals, f_1 and f_2, generate two spurious responses at $2f_1 - f_2$ and at $2f_2 - f_1$. A quick check of the arithmetic will show that these signals will be equally spaced on a linear frequency scale. The amplitude of the spurious signal on the left, $2f_1 - f_2$, is a function of the product of the amplitudes of f_1 squared and f_2; the amplitude of the spurious signal on the right is a function of the product of the amplitudes of f_2 squared and f_1.

This means that if the amplitude of f_1 is increased by 1 dB, the amplitude of $(2f_1 - f_2)$—the spurious signal on the left—increases by 2 dB and the amplitude of $(2f_2 - f_1)$ increases by 1 dB.

A rule can now be formulated that covers the case of two unequal signals. Assume two signals, f_1 and f_2, are at a level of −20 and −29 dBm, respectively, with spurious responses at −96 and −105 dBm, as shown in Fig. 2.

If the amplitude of f_1 is decreased by 3 dB, spurious signal C will be shifted downward by 6 dB. If then f_2 is increased by 6 dB, C will shift upward by 6 dB and be at the same level it was originally; and the amplitude of f_1 and f_2 will be equal (−23 dBm).

The level of two equal signals that yield the same spurious response as the two given unequal signals is determined as follows: Take the stronger of the two signals and subtract from it one-third of the difference between the two signals. This essentially equalizes the two signals, and yields an equivalent amplitude. Two signals at this new level generate the same worst-case third-order spurious level as the two original unequal signals. In the previous example the difference between the two signals is 9 dB [−20 dBm − (−29 dBm)]. The worst spurious is the same as one generated by two equal signals at −20 dBm − 9/3 dB = −23 dBm

Simply subtract one-third of the difference between the two signals from the stronger of the two, find this value on the nomograph, and proceed to find the intercept point and dynamic range in the same manner as for two equal signals. ■ ■

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